

# Towards a Bose glass transition in an optical Penrose quasicrystal

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**Abstract.** – We study numerically a 2D Bose-Einstein condensate in a quasiperiodic array of potential peaks, assumed to be generated by superimposing five blue detuned laser beams. By using a Bogoliubov *ansatz* for the excitations we show that the system approaches a gapless, insulating phase upon increasing the potential, consistent with a Bose glass phase. The characteristics of the transition in terms of phase correlations, oscillatory modes, and superfluid fraction are discussed.

**Introduction.** – The existence of a quasiperiodic structure was first observed in a rapidly quenched metal alloy by Shechtman *et al.* [1]. A sharp peak in the Bragg scattering proved the existence of long range order. The system, however, lacked translational symmetry, undermining the established belief that an ordered structure must be periodic. This novel configuration was generalized by Levine and Steinhardt [2], who formalized the concept of quasiperiodic crystals (QC): these formations exhibit long range order without having translational symmetry, acting both as crystals, admitting a metal-insulator transition, and as amorphous solids, with peculiar localization properties and fractal spectrum [3].

In cold atom research, the investigation of the effects of repulsion between bosons when a disordered potential is present is an ongoing quest. In the late 80s it was speculated [4, 5] that a new phase of matter exists under this condition: The Bose glass (BG). A series of experiments in 1D [6, 7] showed that such a phase exists also for a quasiperiodic potential. This experimental research has been accompanied by a corpus of analytical and numerical articles [8–14] that helped explaining the properties of the glassy phase. In particular, it is predicted that the BG should appear in more than one dimension, and experiments are being done to verify this.

In this Letter we want to investigate the BG phase in a 2D QC lattice. The work of Guidoni *et al* [15, 16] opened up for the possibility of creating an optical QC using an appropriate configuration of lasers. In particular, a configuration with five beams creates a ten-fold symmetric structure, similar to the Penrose tiling [17, 18]. For a non-interacting system, this configuration is known to have

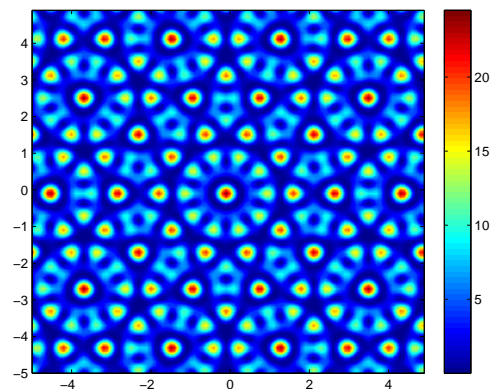


Fig. 1: Penrose potential generated by 5 opposing laser beams, as in Eq. (1) with  $V_0 = 1 E_R$ . In our configuration, the polarization of the laser beams is directed towards the reader.

eigenstates which are neither extended (like Bloch functions), nor exponentially localized. Surprisingly, few works have been done to study the interacting system. Among these, a numerical study by Sanchez-Palencia and Santos [19] has shown that a Penrose-like potential is able to inhibit the diffusion of a BEC. We wish to fill the gap in the literature and show that the quasiperiodic lattice is compatible with a gapless insulating phase, as in the Bose glass.

The optical realization of a QC gives a choice of the sign of the potential: the lattice can be a quasiperiodic array of wells or peaks. We choose the latter, which can be realized with a blue detuned lattice, because in such a

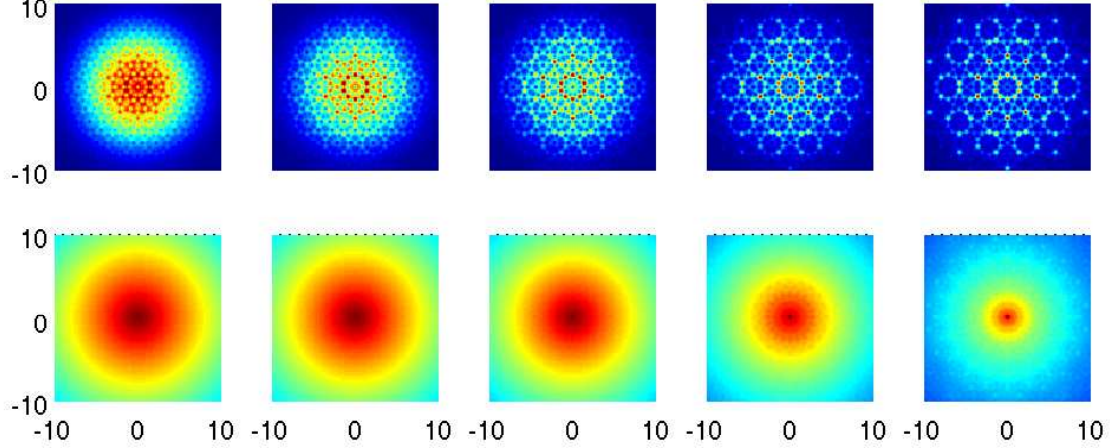


Fig. 2: Condensate density (top row) and spatial average of the correlation function as in Eq (6), for  $g = 10$  with  $V_0 = 1, 2, 3, 4$  and  $4.5 E_R$ . In this colorscale blue is the lowest value, red is the highest.

potential there are no potential wells and a Mott insulating phase can not exist. The possible loss of coherence of the bosons will then depend solely on localization effects due to the quasiperiodicity.

**Penrose tiling.** – A quasiperiodic optical lattice can be generated in 1D by means of superimposing two laser beams with incommensurate wavelengths [6]. In two dimensions it is possible to create a tenfold symmetric quasicrystal structure using five laser beams of equal intensity, lying in the same plane with an angle  $2\pi/5$  between them. The resulting potential is given by the expression [20]

$$V_{QP}(x, y) = V_0 \left| \sum_i \mathcal{E}_i \epsilon_i e^{-i(\mathbf{k}_i \cdot \mathbf{r} + \phi_i)} \right|^2, \quad (1)$$

where  $\mathbf{r} = (x, y)$ ,  $\mathcal{E}_i$  is the relative dimensionless intensity of each laser beam,  $k_i$ ,  $\phi_i$ , and  $\epsilon_i$  are the corresponding wavevector, phase, and polarization.  $V_0$  is an overall intensity. The wavelength  $d = 2\pi/|k|$  of the lattice sets the characteristic length of the system. In the following, we measure all the energies in terms of the recoil energy  $E_R = \hbar^2/m d^2$ , and the unit length is  $d$ . Moreover, we choose  $\mathcal{E}_i = 1$ ,  $\phi_i = 0$ , and the various polarization vectors point all in the same direction, perpendicular to the  $xy$  plane. In Fig. 1 we plot the resulting potential for  $V_0 = 1 E_R$ . This potential appears as a quasiperiodic structure of peaks in the plane.

We consider a quasiperiodic potential combined with a harmonic confinement, so that the external potential in the  $xy$  plane is

$$V(x, y) = V_{QC}(x, y) + \frac{1}{2}\omega^2(x^2 + y^2). \quad (2)$$

**Bogoliubov approach.** – The 2D gas of bosons considered here will in the absence of a quasiperiodic potential form a Bose-Einstein condensate; in the anticipated

Bose-glass phase, long-range coherence will be lost so that the gas can be considered a quasicondensate [21]. In both cases, the system is accurately described by a (quasi-) condensate wavefunction  $\psi_0(x, y)$ , accompanied by quadratic fluctuations described by Bogoliubov modes [22–24]. The (quasi-) condensate wavefunction is governed by the well-known 2D Gross-Pitaevskii equation (GP2D)

$$\frac{1}{2}\nabla^2\psi_0 + g|\psi_0|^2\psi_0 + V\psi_0 = \mu\psi_0. \quad (3)$$

In this 2D approximation, the wavefunction is supposed to be constant in the direction perpendicular to the  $xy$  plane, extending for a length equal to  $a_\perp$ . The value of  $g = g^{(3D)}/a_\perp$  is the scaled interaction strength among the particles in 2D,  $V(x, y)$  is the external potential, and  $\mu$  is the chemical potential. The density of the (quasi-) condensate is given by  $n = |\psi_0|^2$ . Up to second order in the many-body Hamiltonian the atoms outside the (quasi-) condensate occupy excited states which are determined by solving the Bogoliubov equations

$$\begin{aligned} \left(-\frac{1}{2}\nabla^2 + v_j + 2ng - \mu\right) u_j - ngv_j &= \omega_j u_j \\ \left(-\frac{1}{2}\nabla^2 + v_j + 2ng - \mu\right) v_j - ngu_j &= -\omega_j v_j. \end{aligned} \quad (4)$$

where, as usual, the normalization  $\int u_i u_j - v_i v_j = \delta_{ij}$  is enforced. As was found in Ref. [14, 22], the one-body correlation function can be expressed in terms of the Bogoliubov excitations as

$$\begin{aligned} \ln g_1(\mathbf{r}, \mathbf{r}') &= \ln \langle \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(\mathbf{r}') \rangle - \ln \sqrt{n n'} \\ &= -\frac{1}{2} \sum_{j \neq 0} \left\{ \left| \frac{v_j}{\sqrt{n}} - \frac{v'_j}{\sqrt{n'}} \right|^2 \right. \\ &\quad \left. + N_j \left[ \left| \frac{u_j}{\sqrt{n}} - \frac{u'_j}{\sqrt{n'}} \right|^2 + \left| \frac{v_j}{\sqrt{n}} - \frac{v'_j}{\sqrt{n'}} \right|^2 \right] \right\} \end{aligned} \quad (5)$$

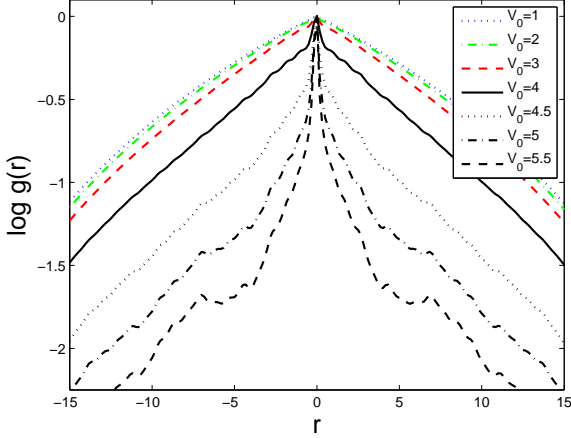


Fig. 3: Logarithm of the rotation average of the correlation function, as in Eq. (8).

provided  $\psi_0$  is real. In this formula,  $\hat{\psi}$  is the many-body boson operator, and for brevity we write  $u_j, v_j, n$  and  $u'_j, v'_j, n'$  instead of  $u_j(\mathbf{r}), v_j(\mathbf{r}), n(\mathbf{r})$  and  $u'_j(\mathbf{r}), v'_j(\mathbf{r}), n'(\mathbf{r})$ .  $N_j$  is the occupation number of the  $j^{\text{th}}$  excited state, as determined by the Bose distribution. In current experiments, this occupation can be made so small that it can be neglected with respect to the depletion from the (quasi-) condensate given by the interaction. In the following we work with  $N_j = 0$  (as for vanishing temperature) and the only contribution to the correlation is the first term in the sum.

We remark that the Bogoliubov approach is a gapless *ansatz* for the excitations in a Bose system. This approach works for the gapless Bose glass phase, but it fails to detect the Mott insulator transition because the latter phase has a gap. The choice of our potential inhibits the formation of a Mott phase, and the only insulating phase in the system is provided by the quasiperiodic pattern of the lattice.

We find the ground state of the GP2D by using an imaginary time evolution with the Fourier split operator method. Subsequently, we diagonalize the Bogoliubov equations by employing the ARPACK libraries with Tchebychev polynomial acceleration [25]. We choose a system size of  $L = 80d$ , and the square grid has 480 points per side. The matrix to diagonalize has therefore the dimensions of  $2 \times 480^2 \sim 4.6 \times 10^5$  points per side. This matrix is not banded, since we have chosen to represent the Laplacian using the Fourier transform. Therefore, in order to have convergence in reasonable time, even with the polynomial acceleration, we must make a cutoff at 100 eigenvectors. We shall see that this cutoff is adequate.

We consider a system with  $g = 10$ ; this relatively weak interaction can be realized with, e.g., a gas of  $\sim 10^5$  atoms, confined in a transverse dimension  $a_\perp \sim 100\text{nm}$ , with a scattering length of  $\sim 10^{-4}\text{nm}$ , which we propose could be realized using Feshbach resonances. The harmonic potential strength in the  $xy$  plane is  $\omega = 8 \times 10^{-2} E_R$ . The

reason for these parameter values is numerical limitations; in principle, there is nothing to prevent the BG transition to occur also for larger values of  $g$ . Also, the numerical results are limited to  $V_0 \leq 5.75 E_R$ : Beyond this parameter regime, an unattainably high spatial resolution was required in order to correctly represent the mode functions.

The top row of Fig. 2 plots the density of the condensate while changing the strength of the lattice from  $V_0 = 1 E_R$  to  $4.5 E_R$ . In the bottom row we plot instead the spatial average of the correlation function [26]

$$g(\mathbf{r}) = \frac{1}{V} \int d\mathbf{R} \langle \hat{\psi}^\dagger(\mathbf{R} + \mathbf{r}) \hat{\psi}(\mathbf{R}) \rangle. \quad (6)$$

This quantity is directly related to what is imaged in experiments; the density after time of flight is to a good approximation equal to the momentum space density  $\rho_k$ ,

$$\rho_k = \frac{1}{V} \int d\mathbf{r} d\mathbf{R} e^{i\mathbf{k}\cdot\mathbf{r}} g(\mathbf{r}). \quad (7)$$

As we can see, for low values of  $V_0$  the characteristic width of  $g(\mathbf{r})$  is the entire condensate, indistinguishable from the average correlation function of a pure condensate in a harmonic potential. When  $V_0 \sim 3 E_R$  this correlation function starts to shrink, and eventually decays rapidly when getting farther from the center.

This behavior is particularly evident when we plot the angular average of  $g(\mathbf{r})$

$$g(r) = \int \frac{d\theta}{2\pi} g(\mathbf{r}, \theta), \quad (8)$$

as is shown in Fig. 3. When a weak lattice is present, the logarithm of the correlation function decays on a scale comparable with the total size of the system. Upon increasing the lattice, a central peak appears where the logarithm of the correlation decays linearly, on a scale shorter than the system size. This is one of the signatures of a crossover to an insulating phase, as reported by Deissler *et al* [7].

**Elementary excitations.** — It is known that the Bose glass phase is caused by low-lying excitations that can flip the phase of the quasicondensate with a little amount of energy. One of the signatures of the glassy phase is the progressive lowering of the excitation spectrum.

In Fig. 4 we plot the energies of the lowest dipole and quadrupole excitations. We see that the raising of the quasiperiodic lattice separates each of the modes into two branches. More importantly, the energy of the excitations drops significantly. These modes can be excited by imposing dipolar and quadrupolar deformations in the confining potential. In particular, a strain can be given to the quasicondensate by a slight change in the harmonic confinement,

$$\frac{1}{2}\omega^2(x^2 + y^2) \rightarrow \frac{1}{2}(\omega + \epsilon)^2 x^2 + \frac{1}{2}(\omega - \epsilon)^2 y^2. \quad (9)$$

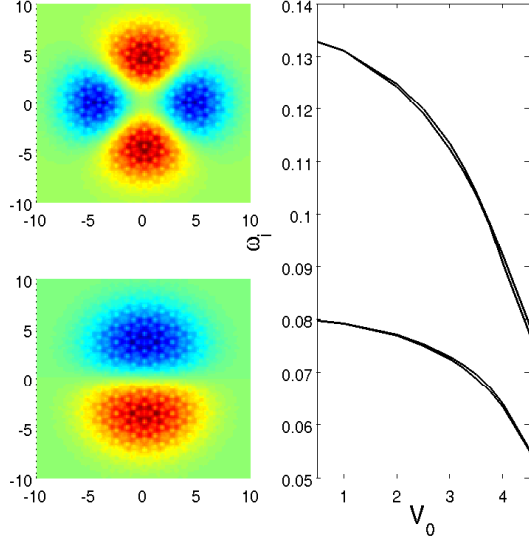


Fig. 4: Dipolar and quadrupolar excitations. Right panel: The four lowest excitation energies, in units of  $E_R = \hbar^2/m d^2$ , as functions of the lattice strength  $V_0$ . Lower left: Bogoliubov amplitude  $v_1$  for the first excited mode; upper left: Bogoliubov amplitude  $v_3$  for the third excited mode, both computed for the case  $V_0 = 1 E_R$ .

Such a deformation will predominantly set off an excitation with an irrotational velocity field. For  $V_0 \sim 0$ , the characteristic frequency of this mode is seen to be  $\tilde{\omega} \approx 0.13 E_R \approx 1.6\omega$ . This is the scissor mode, described in Ref. [27]. Using a hydrodynamical approach, Ref. [27] shows that a superfluid admits quadrupolar excitations (scissor modes) in a harmonic potential, and the lowest frequency for this type of excitation is in the strongly interacting limit equal to  $\sqrt{2}\omega$ ; for our finite value of  $g$ , it is slightly higher at  $V_0 = 0$ . Another important result of Ref. [27] is that a normal fluid would dampen this excitation in a finite time. We argue that the disappearance of the scissor mode provides a way to determine the phase transition to the Bose glass, since when the system is completely normal the quadrupolar excitations are short-lived.

In order to compute the lifetime of the scissor modes one should compute the imaginary part of the stress-tensor-stress-tensor response function. This cannot be done in the simple Bogoliubov approximation, because even for a uniform system the response function involves an integral that is ultraviolet divergent. In order to lay down a microscopic theory of the quadrupolar excitations a renormalization approach seems compulsory, but such a work is outside the scope of the current letter.

**Superfluid fraction.** – The clearest signature of the Bose glass is the fact that the superfluid fraction of the density vanishes, and the system acts as a normal fluid. The superfluid fraction is the subset of the fluid that can move only according an irrotational velocity field, while the rest of the gas constitutes the normal fraction. In-

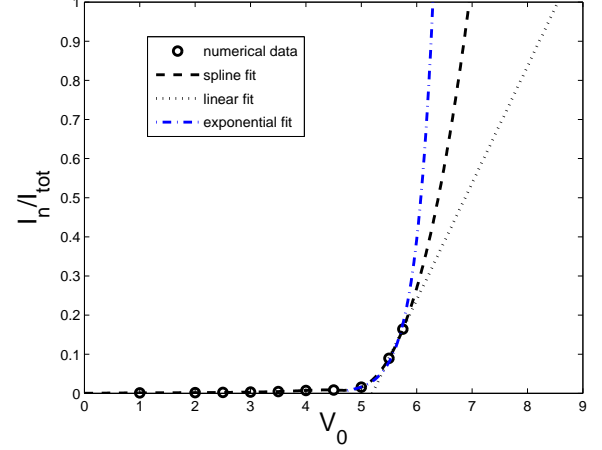


Fig. 5: Normal fraction of the system as function of  $V_0$  (in units of  $E_R = \hbar^2/m d^2$ ), according to Eq. (14). The circles correspond to the ratio  $I/I_{\text{tot}}$  computed according to Eq. (14). In order to estimate where the superfluid part disappears, we plot a spline interpolation of the whole data, a linear fit of the last two points, and an exponential fit of the last three points.

spired by the work in Ref. [28], we compute the normal fraction by looking at the response to a rotation of the system. The external potential can be rotated at a small angular velocity  $\Omega$ . In the rotating frame the Hamiltonian of the system appears as

$$\hat{H}' = \hat{H} - \Omega \hat{L}_z, \quad (10)$$

where the angular momentum operator is (using hats on second-quantized operators but not on first-quantized operators)

$$\hat{L}_z(\mathbf{r}, t) = L_z(\mathbf{r}) \hat{\psi}^\dagger(\mathbf{r}', t') \hat{\psi}(\mathbf{r}, t) \Big|_{\mathbf{r}' \rightarrow \mathbf{r}, t' \rightarrow t}, \quad (11)$$

and  $\mathbf{r} = (x, y)$ , so that  $L_z(\mathbf{r}) = -i \hbar (x \partial_y - y \partial_x)$ .

If the stirring is slow enough, superfluid vortices cannot be generated, and only the normal part takes part in the rotation of the fluid. In the limit of vanishing  $\Omega$ , the increase in energy due to the rotation is given by

$$E_\Omega - E_0 = \frac{1}{2} I_n \Omega^2, \quad (12)$$

where  $I_n$  is the moment of inertia of the normal part. Using standard perturbation theory, the normal moment of inertia is given by second order perturbation theory. Expressed as integrals over the imaginary time  $\tau = -i t$ , it reads

$$I_n = \frac{2}{\beta} \int d\tau_1 d\tau_2 \int d\mathbf{r}_1 d\mathbf{r}_2 \langle \hat{L}_z(\mathbf{r}_1, \tau_1) \hat{L}_z(\mathbf{r}_2, \tau_2) \rangle, \quad (13)$$

with  $\beta = 1/T$ , the inverse of the temperature. Notice that this expression is the  $q = 0, \omega = 0$  limit of the angular-momentum-angular-momentum response function, as described by Pines and Nozières [28]. By applying Wick's

theorem we obtain, in the Bogoliubov approximation

$$I_n = 2 \sum_{i \neq j} \frac{\mathcal{I}_{ij}}{\omega_i + \omega_j} + \sum_i \frac{\mathcal{I}_{ii}}{\omega_i}, \quad (14)$$

with

$$\begin{aligned} \mathcal{I}_{ij} = & \left( \int v_i L_z u_j \right) \left( \int u_j L_z v_i \right) \\ & + \left( \int v_i L_z u_j \right) \left( \int u_i L_z v_j \right), \end{aligned} \quad (15)$$

where the last integrals are only over the spatial coordinates, since the (imaginary) time has been integrated out in the evaluation of the Matsubara frequencies [14]. The normal fraction is given by the ratio  $I_n/I_{\text{tot}}$ , where  $I_{\text{tot}}$  is the total momentum of inertia. This ratio is plotted in Fig. 5. Note that the sum over the excitations is limited by the cutoff in the diagonalization procedure. On the other hand, we checked that this restriction of the number of modes does not alter the result: The highest 10% of the states included were seen to contribute less than 1% of the sum. This is expected, since the Bose glass phase depends on the lowest lying excited modes.

According to our analysis, the normal part starts to increase when the correlation function shows an exponential decay on a scale of the whole system ( $V_0 \sim 5E_R$ ). We cannot see the full transition because no numerical results could not be obtained for  $V_0 > 5.75 E_R$ , as discussed above. However, an assortment of extrapolation methods – Fig. 5 shows results of spline, exponential, and linear extrapolation – indicate that the superfluid part should vanish between  $V_0 = 6 E_R$  and  $V_0 = 9 E_R$ . Moreover, note that the transition to the glassy phase is not sharp, but it appears as a smooth crossover from superfluid to insulator. This effect is due to the finite dimensions of the system, and for the 1D Bose glass it has been experimentally observed in Ref. [7].

**Conclusions.** – We have seen evidence suggesting that a blue detuned optical quasicrystal generates a phase transition in a Bose gas, from a superfluid to an insulating phase. The signatures of this transition are an exponential decay of the correlation function and an increase in the normal part of the gas. Since we have chosen a gapless *ansatz* for the excitations, and a blue detuned lattice without potential wells, we know that this phase is not a Mott insulator. The transition is due to the specific shape of the potential and the localization effects it causes on the Bose gas. Summing up, the quasiperiodic pattern of the lattice leads to a normal, gapless state, compatible with the description of the Bose glass.

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